Acoustic Impedance of Perforated Liners

By definition, the acoustic impedance Z of a fluid medium action on or through a surface of given area A is the ratio of the acoustic pressure at the surface by the volume velocity (acoustic flow rate) at the surface; see Equation 1.

$$Z = \frac{p}{q} = \frac{p}{uA} \qquad (1)$$

Acoustic impedance is useful in the study of acoustic radiation from a surface and the transmission of this radiation thru an acoustic environment. Acoustic impedance is a commonly used tool in the analysis and design of tuned acoustic absorbers, especially acoustic liners.

Impedance can be expressed in either its constituent units (pressure per velocity per area) or in rayls. Specific acoustic impedance z is the ratio of pressure over particle velocity; see Equation 2.

$$z = \frac{p}{u} = ZA \qquad (2)$$

The characteristic impedance of a medium, such as air is a material property defined as

$$z_0 = \rho c \qquad (3)$$

where ρ is the density of the medium, and c is the speed of sound. The characteristic impedance of air at room temperature is about 420 Pa s/m.

The specific acoustic impedance of a single hole

is defined as the pressure difference across the hole over the particle velocity thru the hole, i.e.,

$$z = \frac{p_0 - p}{u_h}$$

where u_h is the particle velocity through the hole and $p_0 - p$ is the pressure difference across the hole; see Figure A-2 in Section A. In view of Equation A-9 of Section A, describing the neck/hole dynamics, the specific impedance of the hole can be written as

$$z = ZA = \frac{p - p_0}{q} A = (Is + B)A = IAs + BA = \rho l' s + BA \qquad (4)$$

Changing the Laplace variable *s* in Equation 4 to $j\omega$ results in the specific acoustic impedance in frequency domain. This frequency dependent, complex specific acoustic impedance is shown in Equation 5

$$z = BA + j\rho l' \omega \qquad (5)$$

The specific acoustic impedance of a single hole is generally made dimensionless by dividing it by the characteristic impedance of its medium z_0 of Equation 3.

$$\varsigma_h = \frac{z}{z_0} = \frac{p_0 - p}{z_0 u_h} = \frac{BA}{\rho c} + j\frac{\omega}{c} l' = R + jkl' \qquad (6)$$

where ς_h is the non-dimensional specific acoustic impedance of a single hole/perforation, $k = \frac{\omega}{c}$ is the wave number and $R = \frac{BA}{\rho c}$ is the dimensionless fluidic damping coefficient (resistance to flow thru the hole).

The non-dimensional specific acoustic impedance of the perforated plate ς_p is defined as

$$\varsigma_p = \frac{p_0 - p}{z_0 u} \tag{7}$$

where *u* is the particle velocity of the medium adjacent to the perforated plate. Using the conservation of mass u_h can be related to *u* as shown in Equation (8)

$$u = \sigma u_h \tag{8}$$

where σ is the porosity[1] of the plate. Thus the perforated plate specific acoustic impedance can be presented as

$$\varsigma_p = \frac{\varsigma_h}{\sigma} = \frac{R + jkl'}{\sigma} \tag{9}$$

The impedance of the plate shown in Equation (9) is commonly used in acoustic analysis of liners.

There exist a number of empirical and semi-empirical relationships for specific acoustic impedance of perforated plates. In the absence of mean flow, Bauer's proposed formulation, Bauer [1977][2], of Equation 10 is commonly used.

$$\begin{aligned} R &= (1 + \frac{l'}{2a})\sqrt{8k\nu / c} \\ l' &= l + 0.25 * (2a) \end{aligned} \tag{10}$$

[1] The ratio of the perforation area by the surface area of the perforated plate

[2] Bauer, B. ''Impedance theory and measurements on porous acoustic liners,'' J. Aircr. **14**, 720–728 ~1977

The two traces in Figure 1 depict the impedance of two perforations with 1/8" and 1/16" hole diameters both having 1.5% porosity and both having a backing spaced 2" behind the perforation. As indicated in Figure 1, and observed in practice, smaller perforation holes shifts the tuning frequency a bit higher and introduces more damping in the perforation.

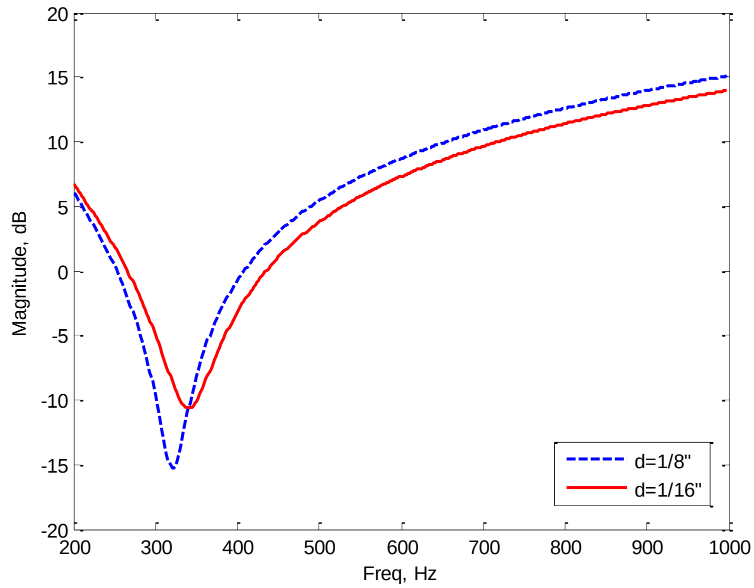

Figure 1 the impedance of two different diameter perforations with 1.5% porosity and a backing space of 2 inches

The use of experimentally measured impedance for a particular perforated plate will normally result in the most accurate modeling results. Note that in absence of mean flow through the hole, the shear layer being created by the acoustic perturbation itself is the main source of absorption/damping.

In presence of mean flow thru the hole and for acoustic perturbations of small amplitude, acoustic-vortex interaction is the main absorption mechanism. Howe[1979][3] built two models to evaluate the amount of energy transferred from acoustic to vortical energy. In his first model, the thickness of the perforation is assumed zero and the Reynolds number thru the hole was assumed large. Moreover, hole spacing-to-radius ratio is considered large (porosity is small) so that apertures do not interact with each other. Subsequently, he modified his model to account for the thickness of the perforation, although the assumption of large Reynolds number and low porosity were left in place.

In Howe's models, the vorticity is postulated to be concentrated in an axi-symmetric vortex sheet separating two regions of potential flow, the jet and the rest of the domain. The vortex sheet being pulsated by acoustic perturbation results in periodical shedding of vortex rings. These shed vortex rings are assumed to have the diameter of the aperture and to be convected at the mean velocity in the hole, *U*. It should be mentioned that a number of researchers, including Hughes & Dowling[1979][4], experimentally verified Hugh's models.

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[3] Howe, M., "On the theory of unsteady high Reynolds number flow through a circular aperture," Proceedings of the Royal Society of London, Series A: Mathematics and Physical Sciences , Vol. 366, 1979, pp. 205–223.

[4] Hughes, I. and Dowling, A., "The absorption of sound by perforated linings," Journal of Fluid Mechanics , Vol. 218, 1990, pp. 299–335.

With the assumptions listed above and assessing the vortex sheet strength by using a Kutta condition, Howe determined the following expression of the Rayleigh conductivity[5] K_R of a hole with the radius a, thickness h, and mean velocity of U

$$K_R = 2a\left(\frac{1}{\gamma - j\delta} + \frac{2}{\pi}\frac{h}{a}\right) \quad \text{with} \quad \gamma - j\delta = 1 + \frac{\frac{\pi}{2} besseli(1,St)e^{-St} + j besselk(1,St)\sinh(St)}{St\left[\frac{\pi}{2} besseli(1,St)e^{-St} - j besselk(1,St)\sinh(St)\right]}$$

where St is the Strouhal number and *besseli* and *besselk* are modified Bessel functions of the first and second kinds. Using the Rayleigh conductivity, the impedance of the perforation[6] can be formulated as $z_p = \frac{j\omega\rho A}{K_R}$, where A is the area of the perforation.

The two traces in Figure 2 depict the impedance of a 1/8" hole diameter, 1.5% porosity perforation with a backing spaced 2" behind the perforation for two mean velocities of U=2.5 m/sec and 5 m/sec. Clear from Figure 2, the impedance corresponding to the higher mean flow has a smoother trough indicating higher acoustic damping in the perforation.

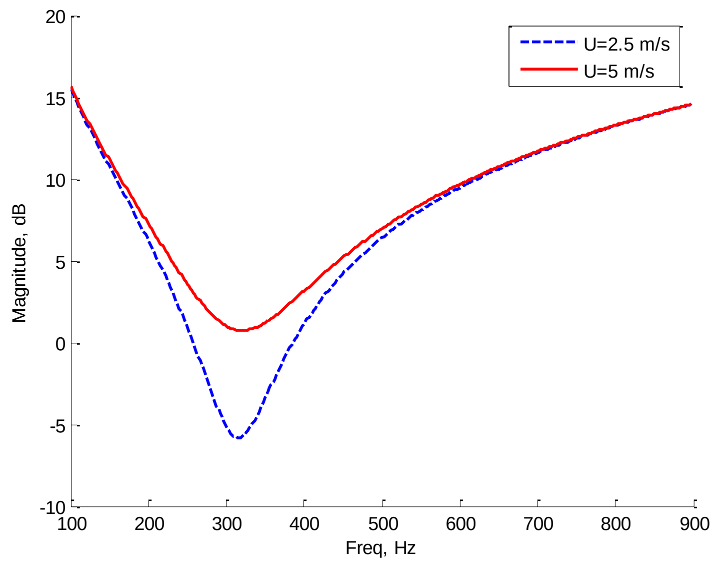

Figure 2 The impedance of a 1/8" hole diameter, 1.5% porosity perforation, and 2" backing at two bias flow velocities

[5] The acoustic behavior of an aperture is described through its Rayleigh conductivity K_R, defined as

$$K_R = \frac{j\omega\rho Q}{(p_+ - p_-)}$$

where ρ is the mean density near the aperture, $\omega = 2\pi f$ is the angular frequency of the pulsating perturbation, Q the amplitude of the flow rate fluctuations through the aperture and p_+, and p_- are the amplitudes of pressure fluctuations, measured below and above the aperture; see Howe [1998].

[6] The definition of impedance of a hole, i.e., $z_h = \frac{(p_+ - p_-)}{V}$, where $V = \frac{Q}{A}$ is the mean velocity of the medium thru the hole, one can formulate the impedance as a function of Rayleigh conductivity , i.e., $z_h = \frac{j\omega\rho A}{K_R}$.