# Tuned Mass Dampers and Vibration Absorbers

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## 1 Tuned Mass Dampers

The use of tuned mass dampers (TMD) is another widely used passive vibration damping treatment. These devices are viscously damped 2nd order systems appended to a vibrating structure. Proper selection of the parameters of these appendages, tunes the TMD to one of the natural frequencies of the underdamped flexible structure, resulting in the addition of damping to that resonance.

Unlike dashpot which is most effective in adding damping to the first mode, TMD can target any mode, including the first, and add considerable amount of damping to it. Another distinction between TMD and dashpot is that TMD is a single point device and can simply be attached to a structure at one end with its other end being free.

To investigate the interaction of tuned mass dampers with a structure, we analyze the vibration of a cantilever beam equipped with one TMD, see Figure 1. The beam is modeled using only 4 modes. The tuned mass damper is appended to the tip of the beam and affects the beam's vibration by inputting a force (control force) to it. The disturbance force is assumed to excite the beam at the tip.

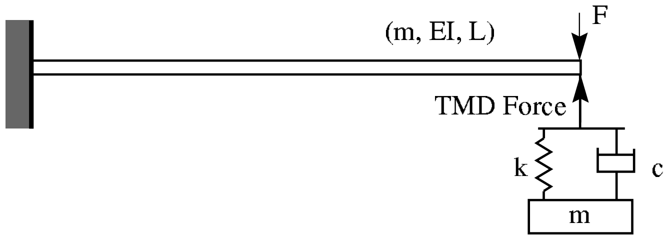


Figure 1: A cantilever beam with a tuned mass damper at the tip

The mass of the TMD and its stiffness are chosen to make the natural frequency of the TMD match to the resonant frequency of the beam to be damped. The damping effectiveness of a TMD is dependent on its damping ratio and the frequency it is tuned at. The lack of large enough damping ratio in the TMD results in breaking the resonant mode, to which is device is tuned for, to two underdamped mode — a phenomenon known as 'mode splitting". A damping ratio between 20 to 45in an effective damping treatment using a TMD. Once the proper damping ratio and the tuning frequency are decided on, and also knowing the mass of the TMD, its spring stiffness and dashpot damping coefficient are evaluated. The FRFs of a cantilever beam without and with the tuned mass damper is shown in Figure 1(a). The first natural frequency of the beam is around 6 Hz. The damping ratio of the TMD is chosen to be 40%. The block diagram of the structure (beam) equipped with a TMD is shown in Figure 1(b).

As indicated in this block diagram, the effect of the TMD on the structure it is appended to would be analogous to that of a feedback controller if the structure was actively controlled with a collocated sensor/actuator arrangement.

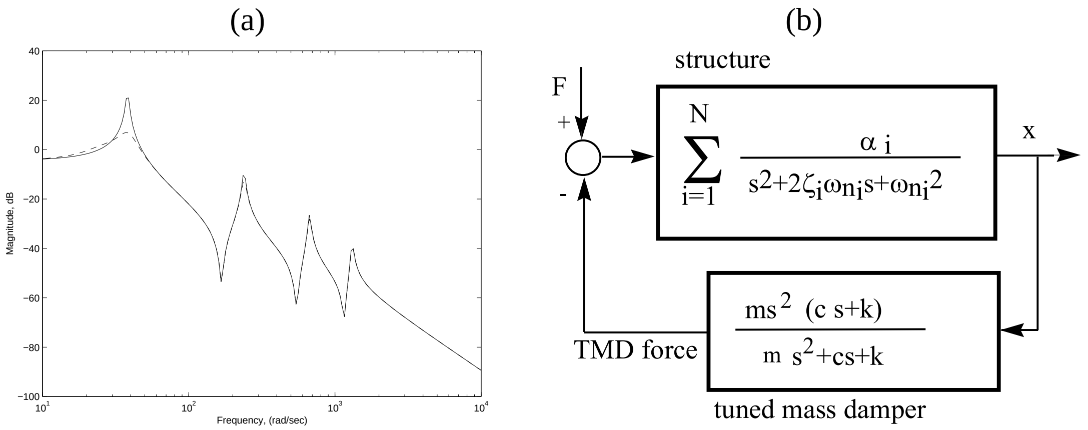


Figure 2: (a) FRFs of cantilever beam (solid line) and the beam equipped with a TMD (dashed line) and (b) block diagram of the beam with a TMD appendage

## The Mechanics of TMD

The frequency response function (FRF) of TMD transfer function is shown in Figure 3. The parameters of the TMD is selected such that, its damped natural frequency is matching the first natural frequency of the beam (40 rad/sec) and its damping coefficient is relatively large. The capability of tuned mass dampers to damp out a particular mode of vibration is due to the 90 degree phase lead that the TMD adds to the open loop system at the natural frequency of that mode (40 rad/sec), along with a gradual change in its phase angle around that frequency.

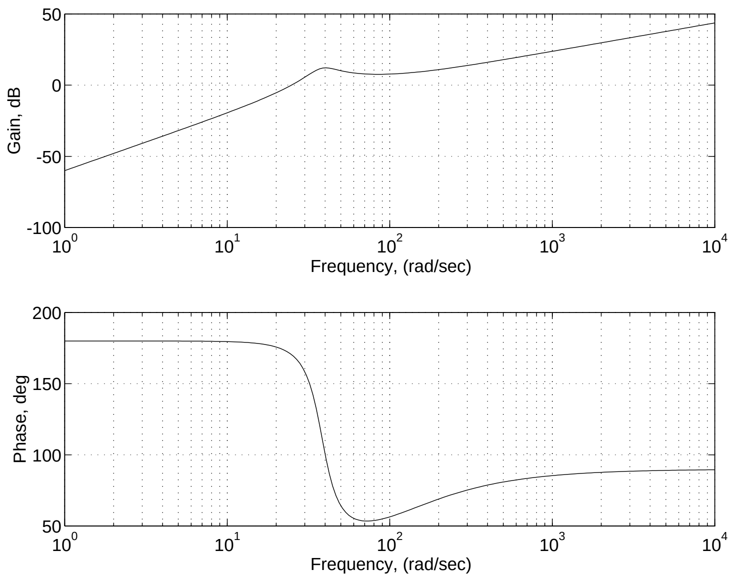


Figure 3: FRF of a tuned mass damper

As seen in Figure 3 and indicated in the block diagram of Figure 1(b), the transfer function of the TMD is not proper. Having the acceleration as the input to the TMD transfer function of Figure 1(b) will take the $s^2$ term out of the numerator of the transfer function circumventing the improperness problem of that transfer function. This makes the realization of this transfer function, electronically or in software, possible leading to an attractive compensation scheme for active vibration control. This attribute will be discussed, in detail, in subsequent chapters on active vibration control.

**Example 1.1:** Appending a TMD on the Unsprung Mass

To be able to use a moderate damping coefficient for the suspension dashpot and yet avoid its undesirable consequence of having an underdamped wheel mode which results in wheel hopping, the use of a TMD has been suggested. Append a TMD to the hub of the suspension system [1] shown in Figure 4, so that damping is added to the wheel mode only. Choosing the mass of the

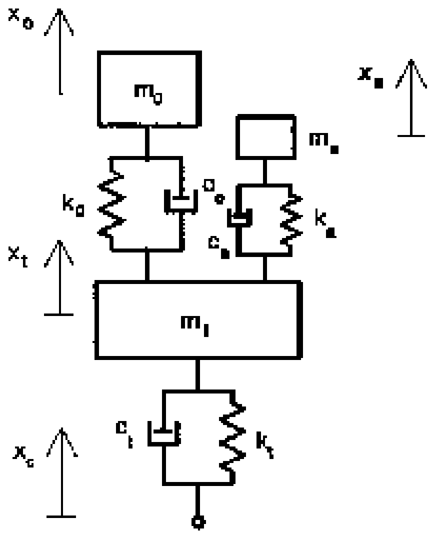

Figure 4: Quarter car suspension model

vibration absorber to be ma=5 kg, tune the absorber to the natural frequency of the wheel mode and study its effects.

The addition of the appendage increases the number of degrees of freedom to 3 (resulting in a 6th ordder system). The equation of motion for this 6th order, 3 DOF system is

$$M_2\ddot{x}+C_2\dot{x}+K_2x = B_{c2}x_{in}$$

where $M_2 = \begin{bmatrix} m_0 & 0 & 0 \\ 0 & m_t & 0 \\ 0 & 0 & m_a \end{bmatrix}$ is the mass matrix, $C_2 = \begin{bmatrix} c_0 & -c_0 & 0 \\ -c_0 & c_0+c_t+c_a & -c_a \\ 0 & -c_a & c_a \end{bmatrix}$ is the damping coefficient matrix, $K_2 = \begin{bmatrix} k_0 & -k_0 & 0 \\ -k_0 & k_0+k_t+k_a & -k_a \\ 0 & -k_a & k_a \end{bmatrix}$ is the stiffness matrix, $B_{c2} = \begin{bmatrix} 0 & c_t & 0 \\ 0 & k_t & 0 \end{bmatrix}^T$ is the excitation matrix, $x = \left\{ \begin{array}{ccc} x_0 & x_t & x_a \end{array} \right\} T$ is the vector of three displacements, and $x_{in} = \left\{ \begin{array}{cc} \dot{x}_c & x_c \end{array} \right\}^T$ is the

[1] $m_0$ is the sprung mass, $m_t$ is the mass of the wheel (tire and its accessories known as unsprung mass), $k_t$ and $c_t$ denote the stiffness and damping of the tire, and $k_0$ and $c_0$ denote the stiffness and damping of the suspension (strut), respectively. $x_0$ and $x_t$ are the displacement of the sprung and unsprung masses, and $x_c$ denotes the road excitation. Using the values $m_t = 40$ Kg, $c_t = 200$ Nsec/m and $k_t = 5e5$ N/m.

excitation vector. Displacement of sprung and unsprung masses in response to the road excitation can be derived in Laplace domain as:

$$X = (M_2 s^2 + C_2 s + K_2)^{-1} B_{c2} \left\{ \begin{array}{c} s \\ 1 \end{array} \right\} X_c \quad (1)$$

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## 2 Vibration Absorbers

As stated earlier, lightly damped inertial appendages, known as 'vibration absorbers', are effective vibration absorption devices. They are mainly used to cancel the structural vibration caused by a single frequency disturbance. To study the vibration cancelation aspect of this passive vibration attenuation device, we append a vibration absorber to a 1 DOF (spring-mass-dashpot) system resembling a flexible structure subject to the disturbance force $F$. Combination of the structure and the vibration absorber results in a 2 DOF system shown in Figure 5. Note that similar to a TMD, a vibration absorber adds an additional degree of freedom to the structure it is appended to. The equation of motion of the system shown in

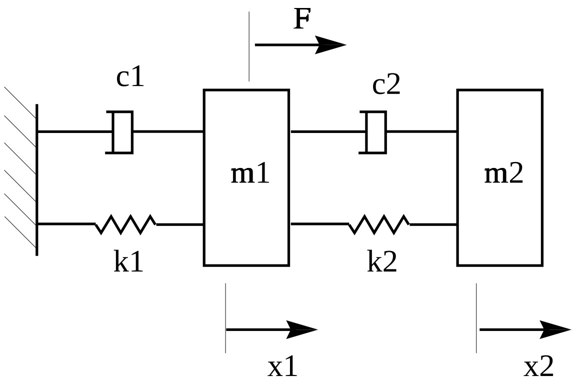


Figure 5: A vibration absorber appended to a second order system

Figure 5 is conveniently expressed in matrix form as

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \left\{ \begin{array}{c} \ddot{x}_1 \\ \ddot{x}_2 \end{array} \right\} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \left\{ \begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \end{array} \right\} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \left\{ \begin{array}{c} x_1 \\ x_2 \end{array} \right\} = \left\{ \begin{array}{c} F \\ 0 \end{array} \right\} \quad (2)$$

In the absence of damping element in the appendage of Figure 5, i.e., $c_2 = 0$, three of the entries of the damping matrix of Equation 2 become zeros resulting in a pair of imaginary zeros[2] in the transfer function mapping the disturbance force $F$ to the structural displacement $x_1$. The location of this zero is on the imaginary axis of the $s$ plane at $\sqrt{k_2/m_2}$ which is the natural frequency of the appendage. When $k_2$ and $m_2$ are selected (appendage is tuned) to match this natural frequency to the disturbance frequency, the structural vibration will be attenuated at that frequency. This is the inner-working of a *vibration absorber* which is in fact a TMD with negligible damping. Vibration absorbers are used primarily for alleviating the forced vibration response of (not adding damping to) the structure.

[2]root of the transfer function numerator

**Example 2.1:** Vibration Absorption in a 1 DOF Aystem
The sinusoidal disturbance force $F$ excites the mass in a single DOF system. Append a vibration absorber to the 1 DOF structure and tune it to cancel the vibration of the mass; see Figure 5.

The FRF mapping the disturbance $F$ to the displacement $x_1$ of the system without and with vibration absorber are depicted in Figure 6. Clear from this figure, the addition of the vibration absorber to the original 1 DOF system ($m_1$, $c_1$, $k_1$) changes the system to a 2 DOF system. Moreover, having very small amount of damping in the vibration absorber results in a deep and sharp zero (anti–resonance frequency) between the two poles (resonance frequencies) of the new 2DOF system. Tuning of this class of vibration

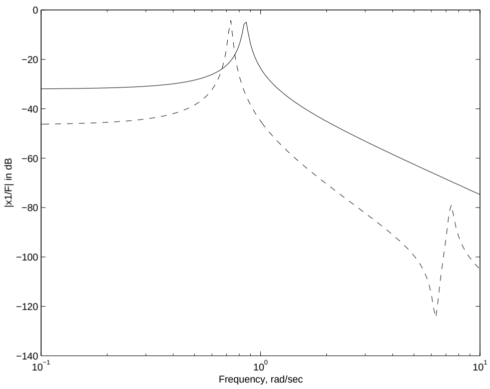


Figure 6: FRF mapping the force input to the mass $m_1$ to the displacement of that mass; 1 DOF system (solid line) and 2 DOF system (dashed line)

absorbers amounts to matching the frequency of the zero shown in Figure 6, which is that same as the natural frequency of the vibration absorver , i.e. $\sqrt(k_2/m_2)$, to the frequency of the disturbance input.

Note that the use of vibration absorbers is effective in reducing forced vibration caused by sinusoidal or at least periodic excitations.

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