Passive Damping Using a Dashpot

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Structural vibration is passively controlled through the modification of the dynamics of the under-damped structure either by increasing damping ratio(s) or altering the order of the structure. The former modification which can be categorized as 'structural damping treatment' enhances the transient vibration characteristics of the structure and is normally done using dashpot(s) and/or viscoelastic material. The latter modification is accomplished by appending inertial devices to the structure. These devices are 2nd order systems with either a solid or liquid inertia. Depending on the amount of damping in them, these devices can either be used for adding damping to a structure or cancelling/absorbing the forced vibration of the structure caused by a disturbance. When the appendage itself has damping ratio of 25 to 45% (known as tuned mass damper) it controls the vibration by adding damping to the structure. And when the appendage has negligible or no damping (known as vibration absorber) vibration is controlled through opposing the disturbance input. The natural frequency of the tuned mass damper is tuned to the resonant frequency that is being damped and the natural frequency of the vibration absorber is tuned to the frequency of the disturbance vibrating the structure.

1 Damping Treatment Using a Dashpot

Dashpot is a commonly used damping element, with automobile suspension as its most notable application. It is a viscous damping element, i.e., the damping force generated by this element is proportional to the net velocity experienced by the element; see Equation 1

$$damping\_force = c\dot{x} \tag{1}$$

where c is the damping coefficient of the dashpot, and x is the next displacement seen by the dashpot. When one end of the dashpot is fixed, it inputs a force to the structure proportional to the velocity of the structure where it is attached resulting in the addition of viscous damping to that structure.

Such damping treatment is demonstrated by attaching a dashpot to the tip of a cantilever beam depicted in Figure 1. The solid line in Figure 2(a) shows the FRF of the beam mapping the disturbance force input at the tip to the displacement output of the tip.

The effect of adding a dashpot to a structure can be modeled by adding a feedback loop feeding the displacement of the structure where the dashpot is located to the structure through the dashpot transfer function cs, where s is the Laplace variable[1]; see Figure 2(b). The dashed line in Figure 2(a) shows the FRFs of the structure equipped with the dashpot. The addition of damping to the beam using dashpot is quite clear.

[1] Note that the dashpot force of Equation 1 $dashpot\_force = [cs]X$, where sX is equivalent to the derivative of displacement x, i.e., velocity $\dot{x}$.

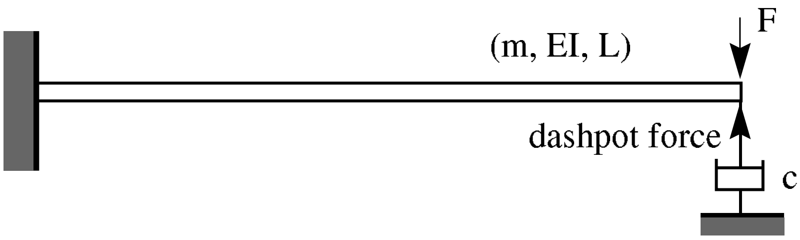

Figure 1: A cantilever beam with a dashpot attached to the tip

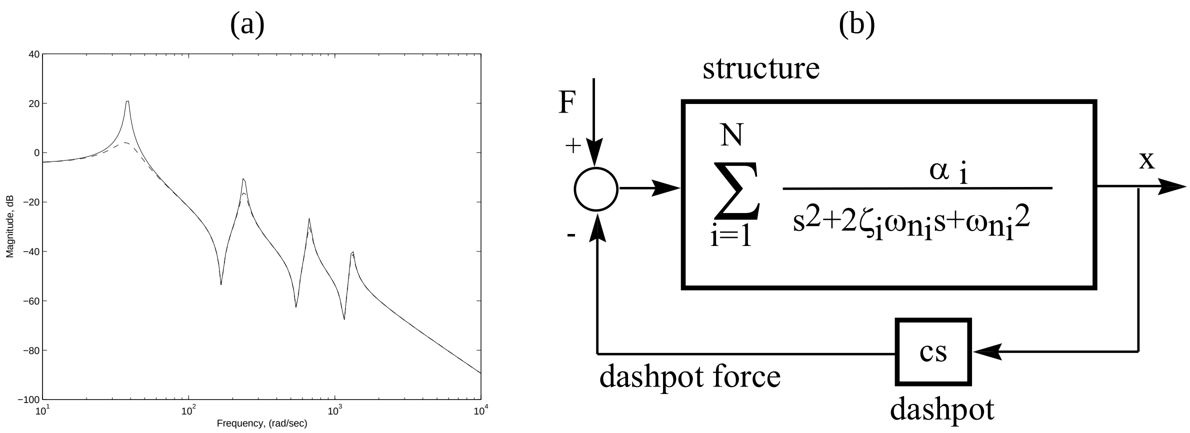

Figure 2: (a) FRFs of cantilever beam (solid line) and the beam equipped with a dashpot (dashed line) and (b) block diagram of the beam with a dashpot

Evident from Figure 2(a), damping treatment of a structure using a dashpot has limited effectiveness on higher order modes, but is most effective on the first mode. This is because the first mode contains most of the vibration energy and dashpot which is an energy dissipater is more effective where there is more energy to dissipate, i.e., the first mode.

Example 1.1: Quarter Car Suspension Model

Consider the quarter car suspension model shown in Figure 3. m_0 is the sprung mass, m_t is the mass of

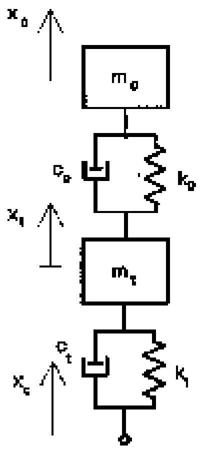

Figure 3: Quarter car suspension model

the wheel (tire and its accessories known as unsprung mass), k_t and c_t denote the stiffness and damping of the tire, and k_0 and c_0 denote the stiffness and damping of the suspension (strut), respectively. x_0 and x_t are the displacement of the sprung and unsprung masses, and x_c denotes the road excitation. Using the values $m_t = 40$ Kg, $c_t = 200$ Nsec/m and $k_t = 5e5$ N/m, plot the frequency response functions mapping road profile to the body (sprung mass) and hub (unsprung mass) displacements. Note that the FRFs exibit two distinct resonances/modes called tire mode and body mode.

The equation of motion for this 4th order, 2 DOF system is

$$M_1\ddot{x} + C_1\dot{x} + K_1 x = B_c x_{in}$$

where $M_1 = \begin{bmatrix} m_0 & 0 \\ 0 & m_t \end{bmatrix}$ is the mass matrix, $C_1 = \begin{bmatrix} c_0 & -c_0 \\ -c_0 & c_0 + c_t \end{bmatrix}$ is the damping matrix, $K_1 = \begin{bmatrix} k_0 & -k_0 \\ -k_0 & k_0 + k_t \end{bmatrix}$ is the stiffness matrix, $B_c = \begin{bmatrix} 0 & 0 \\ c_t & k_t \end{bmatrix}$ is the excitation matrix, $x = [x_0, x_t]^T$ is the vector of two displacements, and $x_{in} = \left\{ \begin{array}{cc} \dot{x}_c & xc \end{array} \right\}^T$ is the excitation vector. Displacement of sprung and unsprung masses in response to the road excitation can be derived in Laplace domain as:

$$X = (M_1 s^2 + C_1 s + K_1)^{-1} B_c X_{in}$$

Note that $X_{in} = \mathcal{L}(x_{in}) = \left\{ \begin{array}{c} s \\ 1 \end{array} \right\} \mathcal{L}(x_c) = \left\{ \begin{array}{c} s \\ 1 \end{array} \right\} X_c$ and $X = \mathcal{L}(x)$, where $\mathcal{L}(\cdot)$ signifies Laplace transformation.

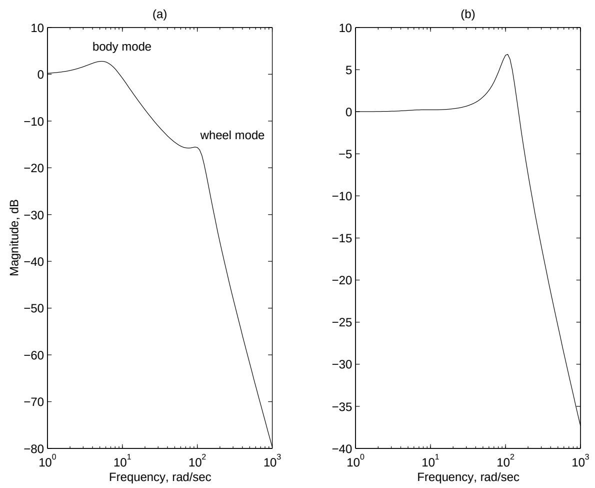

Figure 4: FRFs of sprung (a) and unsprung (b) masses

Example 1.2: Perturbation of Damping Coefficient in a Quarter car Suspension
One of the parameters designers can readily vary to change the characteristicsof the suspension system is the damping coefficient of the dashpot. Using the suspension model derived in Example 1 change c_0 to half and twice as much as the nominal damping coefficient and plot the corresponding magnitude plots of the FRFs.

Figure 5 depicts the magnitude plots of the sprung and usnprung masses. Clear from this figure increase in damping coefficient of the suspension dashpotwill provide good damping for both the tire mode and the body mode damping but at the expense of increased (poor) vibration transmission to the body. On the other hand dashpots with small damping coefficient will make both modes, especially the tire mode, underdamped resulting in excessive resonant vibration of both body and hub. The downside of having low damping on the tire mode is that the tire loses contact with the road resulting in the undesirable loss of stability; the phenomenon is called wheel hopping.

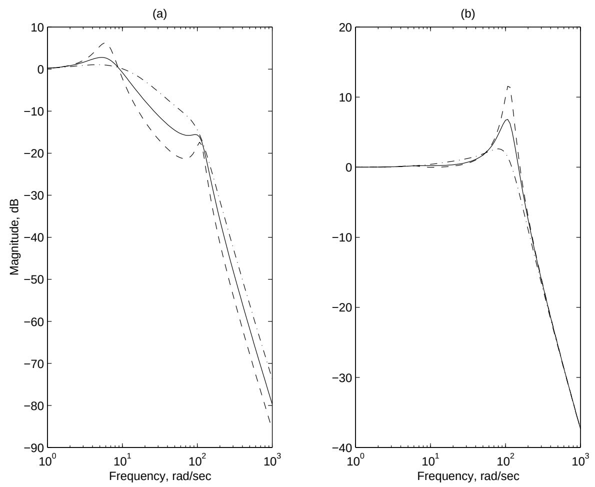

Figure 5: Variations in FRFs of sprung (a) and unsprung (b) masses due to perturbation in damping coefficient; nominal coefficient c_0 (solid line), $c_0/2$ (dashed line), and $2c_0$ (center line)
