Modeling of Flexible Structures

R. Kashani, Ph.D.
www.deicon.com

Depending on the level of detail needed, a flexible structure can be represented and modeled as a simple second order system described by a second order ODE or a set of complex coupled PDE's describing the ∞–order distributed parameter systems. To this effect, the modeling of flexible structures is discussed by analyzing the vibration of a 2–DOF spring–mass–dashpot system and continued describing how continuous systems are described mathematically, using PDE's.

1 Lumped Parameter Flexible Structures

The equation of motion of a 2–DOF discrete system shown in Figure 1 is conveniently expressed by a set

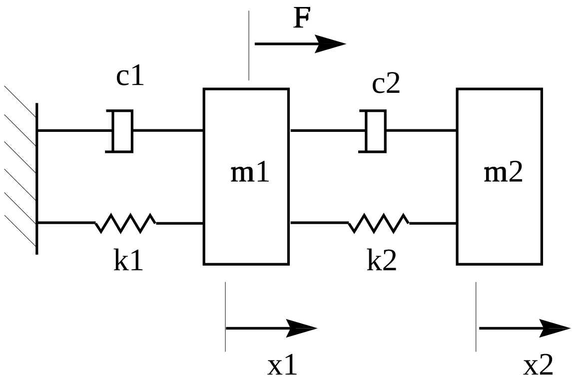

Figure 1: A 2–DOF discrete system

of two 2nd order ODE's, as

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix} \qquad (1)$$

This model, with the generic form of Equation 2 can be extended to describe an n-DOF system.

The model of a linear time–invariant discrete/discretized flexible structure can be expressed by the second order matrix differential equation of

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = u(t) \qquad (2)$$

where M, K, and C are the mass, stiffness, and damping coefficient square, symmetric, matrices with their dimensions equal to the number of degrees of freedom considered, n. u is the forcing function vector.

For systems with classical damping, i.e., the systems with damping matrix C proportional to the mass M and stiffness K matrices, M, K, and C can be diagonalized using the mass normalized orthonormal eigenvectors as the columns of the transformation matrix (see the following section), resulting in

$$\ddot{\eta}_i(t) + 2\zeta_i\omega_i\dot{\eta}_i(t) + \omega_i^2\eta_i(t) = Q_i u(t), \qquad i = 1, 2, \ldots \tag{3}$$

where η_i, ω_i and ζ_i represent the transformed coordinate, natural frequency, and damping ratio of the structure's i-th mode of vibration. When the input is point force, Q_i is the vector of i-th eigenfunction evaluated at the force input location.