Active Vibration Damping using Optimal Control Techniques

R. Kashani, Ph.D.
www.deicon.com

Lead compensation–based controllers described above are reasonably straightforward to synthesize without needing an accurate model for the structure. Although simple and effective, their systematic use is limited to single–input–single output applications. Moreover, they are collocated controllers, i.e., they require the actuation and sensing to occur at the same location(s) which might not be always possible. Lastly, they are not optimal controllers.

The last three decades have brought major developments in the mathematical theory of multivariable feedback systems which include the state space concept for system description and the notion of mathematical optimization for controller synthesis. Various time-domain-based analytical and computational tools have been made possible by these ideas resulting in controller design techniques such as linear quadratic regulation (LQR) and linear quadratic Gaussian (LQG) methods, which their applications to structural control have been studied by researchers. The theory has increasingly concentrated on analytical issues and has not placed enough emphasis on issues which are important and interesting from the perspective of practical design and application. In particular the problem of model uncertainties had been somewhat neglected by these theories.

1 LQR/LQG Control of Structures

LQR is a linear optimal full state feedback control method with the objective of minimizing the impulse response of the states and the control expenditure, in a quadratic sense. It has a very elegant synthesis procedure and possess a high level of stability robustness.

Example 1.1: Active damping of a flexible plate

Design an LQR optimal controller to add damping to a rectangular, 0.08 mm steel, flexible plate, free at two sides and fixed at the other two. The plate is patched with two 10 mil PZT actuators (5cm $\times$ 2cm) and nearly collocated sensors (1.5cm $\times$ 1cm) at two locations; see Figure 1. The two–input–two–output model of the plate containing 6 flexible modes is experimentally identified and available for the design. We assume all the states of the structure is available for feedback.

Weight of 1 is used equally on all the states (modal displacements and velocities). The weight of 10^{-6} was equally applied to the two control actuations. Having the weight matrices decided on, the optimal LQR controller is synthesized. Note that LQR is a full state feedback controller and has 12 inputs and two outputs. Using this controller the closed loop system was formed. The FRFs of the controlled structure are shown in 2.

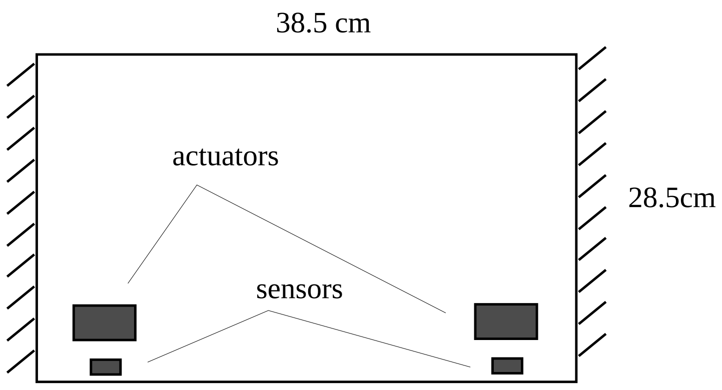

Figure 1: Flexible plate

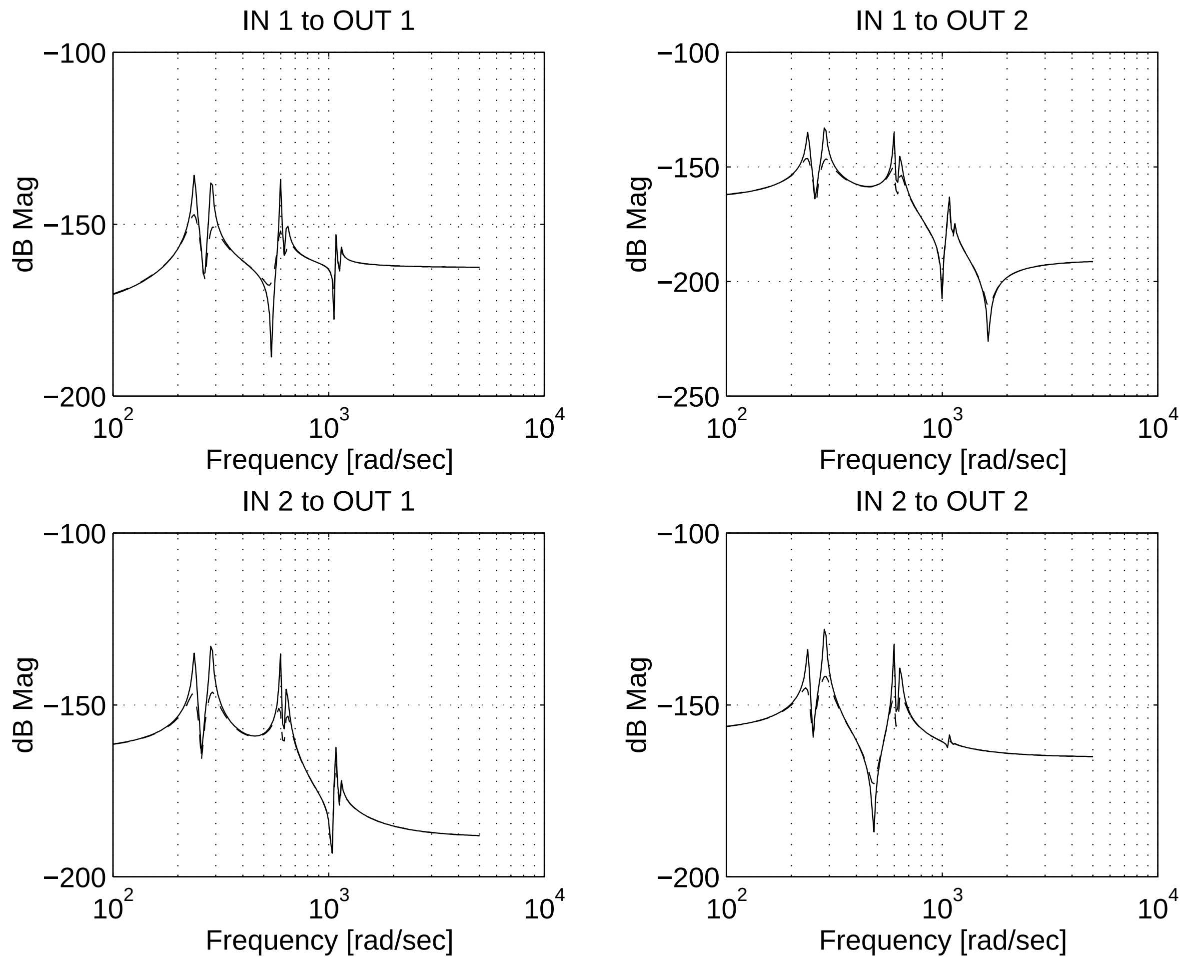

Figure 2: The FRFs of the flexible plate controlled by an LQR; open loop (solid–line) and closed loop (dashed–line)

Except for simple, low DOF, discrete systems, structural states are normally chosen as modal displacements and velocities. These states have only mathematical meaning and thus are not measurable, directly. Not having access to the measured states, for full state feedback, has led to the use of estimators to estimate them. Using a Kalman filter for estimation and feeding the estimated states back is what is known as LQG control. for detail discussion of Kalman filter and LQG control. Covariances of disturbance and measurement noise, are the design parameters of the Kalman filter. In practice, these covariances are not necessarily set to their actual values, but are used as adjustments to tune the filter. For example, one can tighten (increase the accuracy of the filter) the estimation by increasing the process noise covariance.

Example 1.2: Active damping of a flexible plate

Considering that all the states are not normally available for feedback design an LQG optimal controller to add damping to the plate described in Example 1.

The first part of design is similar to the step we took in Example 1, i.e., designing an LQR controller assuming all the states would be available. The next step is designing an estimator, namely Kalman filter, which can estimate the states using the input to the structure (2 strain actuation provided by the piezo patches and measured output from the structure (2 strain measurements). The estimated states produced by this estimator will be used in place of the real states and fed to the LQR controller designed in step 1. Using the LQR design of Example 1 and an estimator designed in this example the FRFs of the closed loop system is constructed and shown in Figure 3.

Unfortunately, the use of estimate of the states leads to the loss of the attractive robustness characteristics of LQR controller. One can vary the level of robustness of the LQG controller by adjusting the disturbance and measurement noise covariances. For example the tighter the estimation, the lower the degree of robustness of the controlled system and inversely the looser the estimation, the more robust the controlled system.

Moreover, LQG are model based controllers and their successful application relies on the existence of an accurate model. This in turn requires the accurate knowledge of the parameters of the model, i.e., natural frequencies, modal damping coefficients, and mode shapes. As stated earlier, the use of truncated models (using only a limited number of modes) leads to observability and controllability spill–overs which result in lack of stability and performance robustness. All these perhaps explains why despite the fact that it has been around for more than 3 decades LQG controller has not been widely used by practitioners.

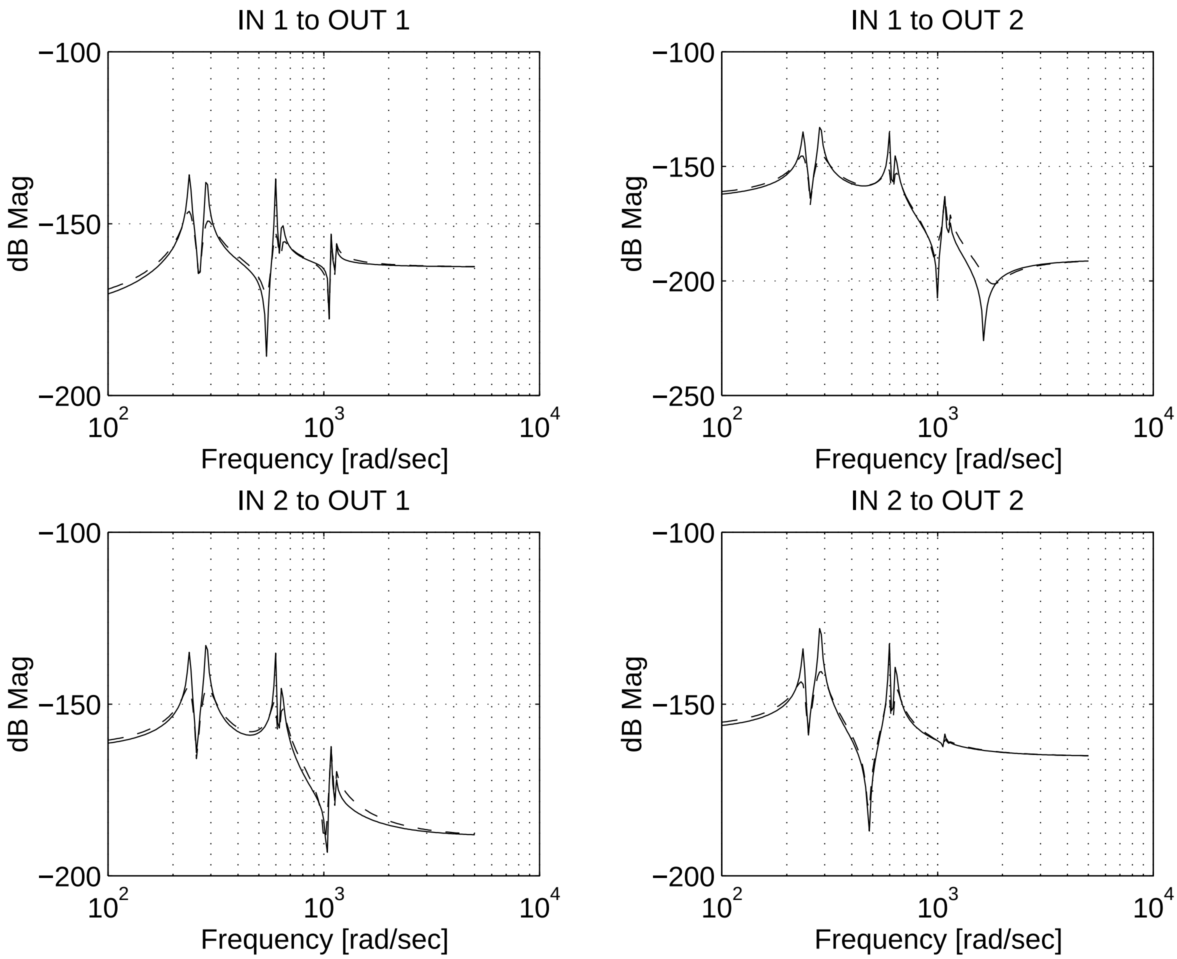

Figure 3: The FRFs of the flexible plate controlled by an LQG controller; open loop (solid–line) and closed loop (dashed–line)